3.10/14

day 43

product

$$f(x) = x\cot^{-1}(x/3)$$

not a simple x

chain rule

$$f'(x) = \frac{d}{dx}(x)\cot^{-1}(x/3) + x\frac{d}{dx}\left(\cot^{-1}(x/3)\right)$$

$$= \cot^{-1}(x/3) + x\left(\frac{-1}{1+\left(\frac{x}{3}\right)^2}\right)\left(\frac{1}{3}\right)$$

$$\cot^{-1}(x/3) - \frac{x}{3\left(1+\left(\frac{x}{3}\right)^2\right)}$$

$$\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{1}{3}\frac{d}{dx}(x) = \frac{1}{3}$$

OR $\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{\frac{d}{dx}(x)(3) - x\left(\frac{d}{dx}(3)\right)}{3^2}$

what if I don't remember $\frac{d}{dx}\left(\cot^{-1}(x)\right)$?

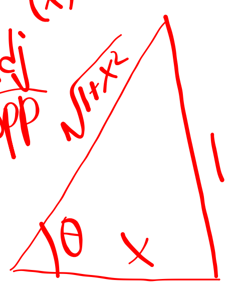
$$\frac{d}{dx}\left(\cot\left(\cot^{-1}(x)\right) = x\right)$$

$$-\csc^2\left(\cot^{-1}(x)\right)\cdot\frac{d}{dx}\left(\cot^{-1}(x)\right) = 1$$

$$\text{so } \frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{\sin^2\left(\cot^{-1}(x)\right)}$$

$$\theta = \cot^{-1}(x)$$

$$\cot x = \frac{adj}{opp}$$



$$\sin\theta = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{-1}{\left(\sqrt{1+x^2}\right)^2} = \frac{-1}{1+x^2}$$

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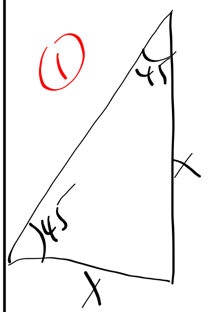
day 43

3.11/7) isosceles right triangle



① a) know legs; want $\frac{dA}{dt}$

② $A = \frac{1}{2}x \cdot x = \frac{1}{2}x^2$

③ $\frac{dA}{dt} = x \cdot \frac{dx}{dt}$ take deriv wrt t

④ subs in.... $\frac{dx}{dt} = +2$ m/sec

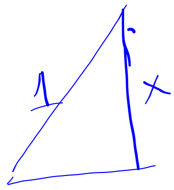
$x = 2m$

$\frac{dA}{dt} = 2(2) = 4\ m^2/sec$

b) $A = \frac{1}{2}x^2$; $\frac{dA}{dt} = x\frac{dx}{dt}$

④ know $\frac{dx}{dt} = 2$ m/sec

hypotenuse = 1m



$1^2 = x^2 + x^2 = 2x^2$

$\frac{1}{2} = x^2$

$\frac{1}{\sqrt{2}} = \sqrt{\frac{1}{2}} = x$

$\frac{dA}{dt} = \frac{1}{\sqrt{2}}(2)\frac{m^2}{sec}$

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$\frac{2}{\sqrt{2}} = \frac{2^1}{2^{1/2}} = 2^{1-\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

$\frac{\sqrt{2} \cdot \sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\sqrt{2} = \frac{2}{\sqrt{2}}$

3.11/7c have $\frac{dx}{dt} = 2$ m/sec want $\frac{d(\text{hypotenuse})}{t}$

c = hypotenuse

② $c^2 = x^2 + x^2 \Rightarrow c^2 = 2x^2$ → Poop

$c = \sqrt{2x^2}$

$= \sqrt{2} \cdot x$

$2c\frac{dc}{dt} = 4x\frac{dx}{dt}$

$\frac{dc}{dt} = \frac{4x\frac{dx}{dt}}{2c} = \frac{2x\frac{dx}{dt}}{c}$

$= 2x\frac{dx}{dt}{=2} = \sqrt{2}(2) =$

day 43

3.10/37) $f(x) = 3x+4$ at $(16,4)$

$(4,16)$ $f^{-1}(x)$

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

$16 = 3x+4$

$4 = x$

$$\frac{d}{dx}\left(f^{-1}(x)\right)\Big|_{(16,4)} = \frac{1}{f'\left(f^{-1}(16)\right)} = \frac{1}{f'(4)}$$

$$f'(x) = 3 \longrightarrow = \frac{1}{3}$$

$y = 3x+4$

$x = 3y+4$

$y = \frac{x-4}{3}$

$y' = \frac{1}{3}$

what if I don't remember that

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}?$$

start with

$$f\left(f^{-1}(x)\right) = x$$

$$f'\left(f^{-1}(x)\right)\cdot\frac{d}{dx}\left(f^{-1}(x)\right) = 1$$

so $\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}$

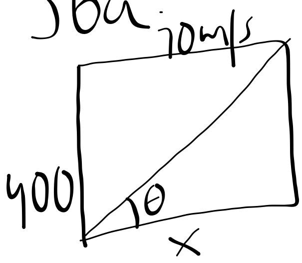
what if I don't remember where to start....

$$f^{-1}(f(x)) = x$$

$$\frac{d}{dx}\left(f^{-1}\right)(f(x))\cdot f'(x) = 1$$

day 43

36a.



$\tan^{-1}(x)$

$$\frac{d\theta}{dx} = \boxed{\frac{1}{1+x^2}} = \frac{1}{1+(400/x)^2}\left(\frac{d}{dx}\frac{400}{x}\right)$$

$$= \frac{1}{1+(400/x)^2}\left(\frac{-400}{x^2}\right) = \frac{-400}{x^2+160,000}$$

$$\theta = \tan^{-1}\left(\frac{400}{x}\right)$$

$$= \frac{-400}{(500)^2+160,000} = 9.75\times10^{-4}\ \frac{\theta}{s}$$

$\frac{\text{radians}}{m}$

day 43

3.9/78 Bob has Q

$$f(x) = \ln\left(\frac{2x}{(x^2+1)^3}\right)$$

"desired" method

$$f(x) = \ln(2x) - \ln\left((x^2+1)^3\right)$$
$$= \ln 2 + \ln x - 3\ln(x^2+1)$$

& take deriv now:

$$f'(x) = \frac{1}{x} - 3\left(\frac{1}{x^2+1}\right)(2x)$$
$$= \frac{1}{x} - \frac{6x}{x^2+1}$$

longer, more direct, way

$$f'(x) = \frac{(x^2+1)^3}{2x} \cdot \frac{d}{dx}\left(\frac{2x}{(x^2+1)^3}\right)$$

$$f'(x) = \frac{(x^2+1)^3}{2x}\left[\frac{2(x^2+1)^3 - 2x(3(x^2+1)^2 2x)}{(x^2+1)^6}\right]$$

$$f'(x) = \frac{(x^2+1)^3}{2}\left[\frac{2(x^2+1)^3 - 12x^2(x^2+1)^2}{(x^2+1)^6}\right]$$

$$f'(x) = \frac{(x^2+1)^5}{2(x^2+1)^6}\left[2(x^2+1) - 12x^2\right]$$

$$= \frac{x^2+1-6x^2}{x^2+1}$$

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day 43

$\underline{3.10/5}$ s'pose f is $\boxed{\text{one-to-one}}$

$f(2)=8$ Key: $\Rightarrow f^{-1}(8)=2$

$f'(2)=4$

what is $(f^{-1})'(8) = (f^{-1})'(x)\Big|_{x=8}$

Recall (or Re-figure out)

$f'(x)\Big|_{x=8} = \dfrac{1}{f'(f^{-1}(x))}\Big|_{x=8} = \dfrac{1}{f'(f^{-1}(8))} = \dfrac{1}{f'(2)}$

$= \dfrac{1}{4}$

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